

gmath, an Excel add-in for gnomonics

This document is for version 20180304 by Steve Lelievre

Summary

The *gmath* add-in extends Microsoft Excel with functions for trigonometry using degrees; converting angles between decimal angles and degrees, minutes and seconds; and determining solar data such as Equation of Time (EoT) or Solar Declination (SD) for a numbered day of the year or for a specific date and time. Many of the functions use methods from Jean Meeus' book *Astronomical Algorithms* (1998). Installing *gmath* is a one-time exercise after which the functions are available automatically whenever you use Excel.

gmath is not a library of sundial generator functions. For example, there is no function that uses latitude and hour angle to calculate a shadow angle of a horizontal dial. Rather, the aim is to make designing dialing spreadsheets a little faster and more convenient – but leaving the gnomonic principles in the hands of the designer.

Figs. 1 and 2 show example calculations for a horizontal dial using standard Excel functions and using *gmath* functions. The applicable formulae are also show as text.

These examples show how one advantage of *gmath* comes from the shorter formulae and simpler bracketing needed.

A	B	C
Horizontal dial calculation		
Latitude of Palmerston North, NZ		
3	Degrees	-40
4	Minutes	21
5	Seconds	8.3
6	As radians	<code>RADIANS((ABS(D3)+D4/60+D5/3600)*SIGN(D3))</code>
7		-0.704280593
8	Hour Angle	Hour Line Angle
9	0	<code>DEGREES(ATAN(SIN(SC56)*TAN(RADIANS(A9))))</code>
10	15	<code>DEGREES(ATAN(SIN(SC56)*TAN(RADIANS(A10))))</code>
11	30	<code>DEGREES(ATAN(SIN(SC56)*TAN(RADIANS(A11))))</code>
12	45	<code>DEGREES(ATAN(SIN(SC56)*TAN(RADIANS(A12))))</code>
13	60	<code>DEGREES(ATAN(SIN(SC56)*TAN(RADIANS(A13))))</code>
14	75	<code>DEGREES(ATAN(SIN(SC56)*TAN(RADIANS(A14))))</code>
15	90	<code>DEGREES(ATAN(SIN(SC56)*TAN(RADIANS(A15))))</code>

Figure 1. Horizontal Dial Calculation - Standard

A	B	C
Horizontal dial calculation		
Latitude of Palmerston North, NZ		
3	Degrees	-40
4	Minutes	21
5	Seconds	8.3
6	As decimal degrees	<code>GANGLE(C3,C4,C5)</code>
7		-40.35230556
8	Hour Angle	Hour Line Angle
9	0	<code>GATAN(GSIN(SC56)*GTAN(A9))</code>
10	15	<code>GATAN(GSIN(SC56)*GTAN(A10))</code>
11	30	<code>GATAN(GSIN(SC56)*GTAN(A11))</code>
12	45	<code>GATAN(GSIN(SC56)*GTAN(A12))</code>
13	60	<code>GATAN(GSIN(SC56)*GTAN(A13))</code>
14	75	<code>GATAN(GSIN(SC56)*GTAN(A14))</code>
15	90	<code>GATAN(GSIN(SC56)*GTAN(A15))</code>

Figure 1. Horizontal dial calculation - *gmath*

The *gmath* functions

Trigonometry using decimal degrees	
<code>gsin(angle)</code>	Sine
<code>gcos(angle)</code>	Cosine
<code>gtan(angle)</code>	Tangent
<code>gcot (angle)¹</code>	Cotangent
<code>gscn (angle)^{1,2}</code>	Secant
<code>gcsc(angle)¹</code>	Cosecant
<code>gasin(value)</code>	Arcsine, or inverse sine
<code>gacos(value)</code>	Arccosine
<code>gatan(value)</code>	Arctangent
<code>gatan2(x, y)</code>	Performs the software function atan2 to give an angle
<code>gacot(value)¹</code>	Arccotangent

¹ Not available for Excel 2010 or earlier.

² The name of this function is not consistent with the Excel equivalent, sec. I chose not to call it *gsec* because that name would be too close to my *gsecs* function.

Solar data	
gobl()	Returns a medial value for obliquity of the elliptic. Specifically, it returns 23.43177307652°.
gobled(date)	Calculates the actual obliquity for a specific Excel Date value (i.e., at a date and time).
gdcl(day)	Returns a medial value for the declination of the sun at noon on a chosen day of the year. For the purposes of <i>gdcl</i> there are 366 days in a year, numbered from 1 for January 1 st .
gdclcd(date)	Calculates the actual declination of the sun for a specific Excel Date value.
geot(day)	Returns a medial value for the Equation of Time at noon on a chosen day of the year. The returned value is in decimal degrees i.e. an Hour Angle. For the purposes of <i>geot</i> there are 366 days in a year, numbered from 1 for January 1 st .
geoted(date)	Calculates the actual Equation of Time for a specific Excel Date value. The returned value is in decimal degrees, i.e. as an Hour Angle adjustment.
Miscellaneous	
gmathver()	Returns a version statement as a string of characters.
grefract(altitude, pressure, temperature)	Estimates the apparent altitude of the sun from a true altitude, using Bennett's approximation for refraction. The inputs for atmospheric pressure and air temperature are optional, defaulting to 1010 millibars and 10°C respectively.
gunrefract(altitude, pressure, temperature)	Estimates the true solar altitude from an apparent altitude, using Sæmundsson's approximation for refraction. The inputs for atmospheric pressure and air temperature are optional, defaulting to 1010 millibars and 10°C respectively.
Angle conversion	
gangle(degees, minutes, seconds)	Converts an angle expressed as degrees, minutes and seconds, to decimal degrees. Each input is optional, defaulting to zero.
gdegs(angle) gmins(angle) gsecs(angle)	Used for converting decimal degrees back to degrees, minutes and seconds. <i>gdegs</i> , <i>gmins</i> and <i>gsecs</i> extract the parts of an angle after rounding to the nearest whole second. The rounding process may cause 60 seconds to be carried back as a minute, or 60 minutes as a degree. Thus the results are only meaningful if all 3 function are used together to decompose an angle.
gpart(angle, part, mode, precision)	Extracts the specified part of an angle with rounding of the least significant part requested, to the specified precision. Used for converting decimal degrees back to degrees, minutes and seconds when the preceding three functions are not applicable. See the explanation given later.
greduce(angle)	Reduces an angle to the range $-180 < \text{angle} \leq 180$. This may result in a change of sign. For instance, -185° is converted to $+175^\circ$. There is no return value of exactly -180° ; it is converted to $+180^\circ$.

Using *gmath*

See the later section for installation instructions. Once *gmath* is installed, you use its functions just like any of Excel's native functions. You should never need to open or edit the *gmath.xlam* file. Instead, simply open a new or existing Excel workbook (.xlsx file) and build your spreadsheets as required.

As with any Excel function, *gmath* allows absolute or relative cell references, as well as fixed values, to be used as inputs. Excel native functions and *gmath* functions can be combined in a cell formula.

In *gmath*, angles are treated as decimal degrees. Except where indicated in the function definitions above, inputs and return values are double precision numbers.

Solar data returned by *gmath* functions are for the Prime Meridian. If a local value is needed you must adjust accordingly, prorating 1 day of the Earth's rotation for 360 degrees of longitude.

Some of the solar data functions take a numbered day of year, counted from 1 to 366, as the input. In this arrangement, the day numbers on which the months start are 1, 32, 61, 92, 122, 153, 183, 214, 245, 275, 306, and 336. February 29th, the intercalary day, is day 60.

Precision and significant digits

Although angles are stored and processed by *gmath* using variable type Double, in practice there is usually little benefit from showing calculated angles at a precision of more than 2 or 3 decimal places.

For example, the fourth decimal place of a decimal degree corresponds to less than 3 seconds of arc; for geographic coordinates it represents less than 100 metres even at the equator. Similarly for shadow angles, penumbral effects limit the precision attainable to several minutes.

Comments on how Excel displays numbers

Confusion may arise because Excel's default cell display format, *General*, varies the number of digits shown depending on the column width available.

Even if you set the display format to *Number* it may still be misleading - cell values are displayed using 2 decimal places unless explicitly changed. For example, a cell containing a value of 59.9999 will display as 60.00 if the default number of decimal places is used. My recommendation is to set cells containing numbers to *Number* format and then to adjust the column widths and the number of decimal digits displayed to match the precision required. See figs. 3 and 4.



Figure 3. Mismatch between cell content and displayed value in General format; Use the applicable dropdown menu to change the cell's format.

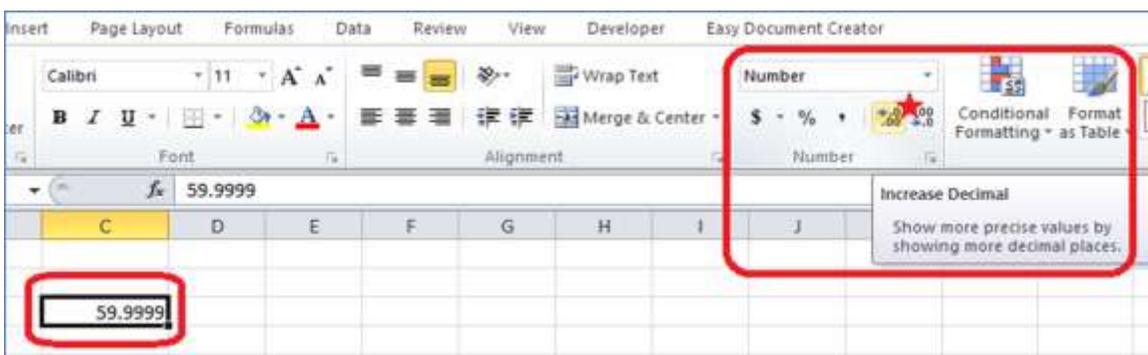


Figure 4. Setting the number of digits displayed to match the cell content's precision is achieved using the controls highlighted by a red asterisk.

Notes on the solar data functions

For the functions that return medial values for a numbered day of the year, returned values are the average of the maximum and minimum values occurring over the years 2020 to 2099, for the Prime Meridian. I chose this range of years on the basis of 80 years as a reasonable upper limit for the lifespan of a new dial. The medial values never vary from daily values by more than 19" for Obliquity, 15.55' for SD, and the equivalent of 14.33 seconds of time for EoT, over the entire period.

Meeus' algorithms make use of Julian Day numbers. Excel has its own internal date representation, equivalent to a Julian Day but with a different starting point in history. When applicable (i.e. for functions whose name ends 'ed' for Excel Date) *gmath* makes the required adjustment. Dates and times provided as inputs to *gmath* should be Excel Dates. Dates before January 1st, 1900 are not handled by Excel.

⚠ An Excel Date is actually a date and time. If you're using *gmath* functions to obtain a solar value for a specific date be sure to specify the time of day required (usually noon) as well as the date.

The EoT functions return angles, for convenient adjustment of Hour Angles. If minutes and seconds of time are required, divide the returned value by 15 and then use the *gmath* angle conversions functions: to express the angle as minutes and whole seconds you would use *gmins*(angle / 15) and *gsecs*(angle / 15).

Pay attention to sign - *gmath* returns values that must be subtracted from apparent time to get mean time.

Notes on the miscellaneous functions

Although atmospheric refraction is rarely considered in sundial design, two functions are included for completeness. *grefract* adjusts a true altitude to an apparent altitude; *gunrefract* does the converse.

Atmospheric Refraction is significant if the sun is close to the horizon. It is usually ignored, but for precise calculations of day length, sunrise or sunset one might take it into account. If doing so, bear in mind that the *gmath* functions return approximations. There is no allowance for the elevation of the place, nor how topography affects the timing of the sun's appearance or disappearance from view, nor localized atmospheric effects, nor wavelength of light. Meeus warns that studies indicate that the actual refractive effect encountered at the horizon routinely deviates by as much as 0.3° from predicted values (p. 107).

Notes on the angle conversion functions

General observations

These functions return #VALUE if an input is out of range or an invalid combination of inputs is detected. Certain invalid inputs or combinations of inputs may not be detected, so users should check that the results of calculations appear reasonable. As well, other Excel error codes may be encountered if generated by the underlying functions used by *gmath*.

Excel's in-built rounding mechanism follows the "round half down" convention, otherwise known as "Banker's Rounding". *gmath* uses the "round half up" convention commonly used in scientific work.

gangle(*degees*, *minutes*, *seconds*)

Degrees may be positive or negative. Minutes and seconds must be less than 60, and normally are positive matching how angles are conventionally written; for example, *gangle*(-30, 30) converts to the decimal angle -30.5°.

Non-integer input values are accepted. For example, *gangle*(10, 15.5) evaluates to the same angle as *gangle*(10, 15, 30) does.

If degrees is not given or 0, minutes may be negative; if neither degrees nor minutes are given or both are zero, seconds may be negative, e.g. *gangle*(, -30, 0) converts to -0.5°.

gdegs, gmins and gsecs

Use these functions together to convert from decimal degrees to a whole number of degrees, a whole number of minutes and a whole number of seconds. Because they perform rounding, the functions are meaningful only if all three parts of an angle are extracted. To see why, consider a calculation that produces 59.999886° , equivalent to $59^\circ 59' 59.99''$, as the result. With rounding, the seconds become 60, which become 0 with an extra minute carried back. In turn, the minutes value becomes 60 which becomes 0 with an extra degree carried back. Thus the return values are *gdegs*: 60, *gmins*: 0, *gsecs*: 0. If this is not the desired conversion use *gpart* instead.

For an example of a case requiring the switch to *gpart*, consider the conversion of 59.999719° , which is $59^\circ 59' 58.99''$, and assume you wish to display the angle as whole degrees and whole minutes with no seconds shown. Returned values would be *gdegs*: 59, *gmins*: 59, *gsecs*: 59. Expressing the angle using only whole degrees and whole minutes isn't just a matter of skipping the *gsecs* value - doing so would treat the angle as $59^\circ 59'$ instead of its true closest degrees and minutes value, $60^\circ 0'$. Instead, use *gpart*(59.999719, "d", "dm", 0) and *gpart*(59.999719, "m", "dm", 0), described below.

Negative angles close to zero degrees are handled by returning a negative value from *gmins* or *gsecs* as applicable. If this is not the desired behaviour, use the absolute value of the angle as the input in conjunction with Excel's *sign* function.

gpart(angle, part, mode, precision)

The *gpart* function generalizes *gdegs*, *gmins* and *gsecs*. The Part parameter determines the element to be returned; allowed values are "d", "m" and "s". The Mode parameter controls how the angle is to be treated – as degrees, as degrees and minutes, or as degrees and minutes and seconds; allowed values are "d", "dm" and "dms". The Precision parameter is an integer specifying the decimal digits for the last applicable element. The following table shows the combinations that can be used.

Parts required	Function calls
Angle rounded to the nearest whole degree	<i>gpart</i> (angle, "d", "d", 0)
Decimal degrees to n decimal places ³	<i>gpart</i> (angle, "d", "d", n)
Degrees and nearest whole minute	<i>gpart</i> (angle, "d", "dm", 0)
	<i>gpart</i> (angle, "m", "dm", 0)
Degrees and decimal minutes to n decimal places	<i>gpart</i> (angle, "d", "dm", n)
	<i>gpart</i> (angle, "m", "dm", n)
Degrees, minutes and nearest whole second	<i>gpart</i> (angle, "d", "dms", 0) ⇔ <i>gdegs</i>
	<i>gpart</i> (angle, "m", "dms", 0) ⇔ <i>gmins</i>
	<i>gpart</i> (angle, "s", "dms", 0) ⇔ <i>gsecs</i>
Degrees, minutes and decimal seconds to n decimal places	<i>gpart</i> (angle, "d", "dms", n)
	<i>gpart</i> (angle, "m", "dms", n)
	<i>gpart</i> (angle, "s", "dms", n)

Negative angles close to zero degrees are handled by returning a negative minutes or seconds value as applicable. If this is not the desired behaviour use the absolute value of the angle as the input, in conjunction with Excel's *SIGN* function.

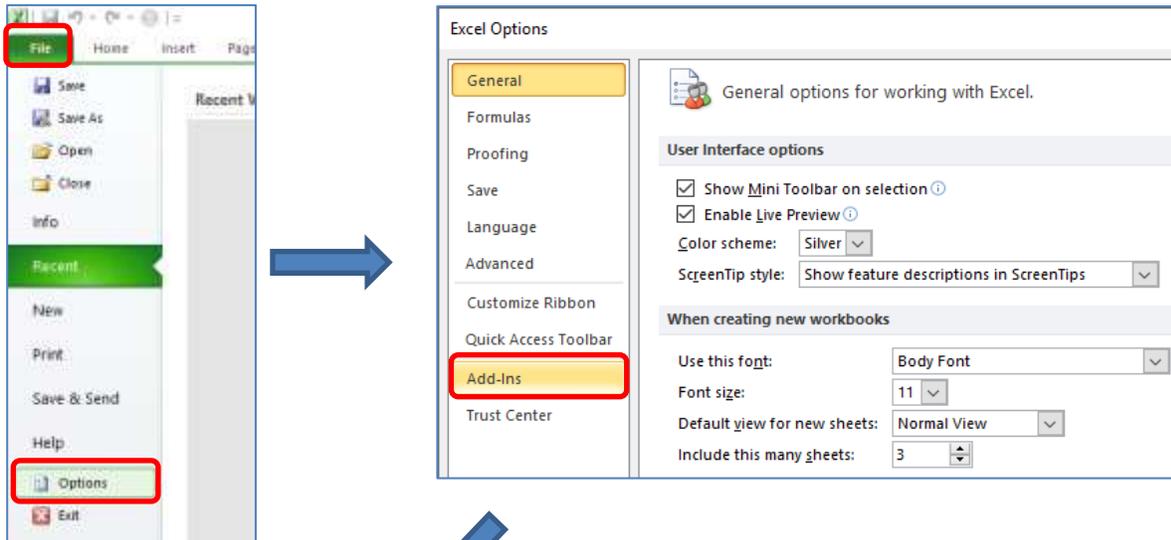
³ This may appear to be equivalent to rounding an angle by using Excel's built-in rounding function. However, that function uses so-called "Bankers' Rounding" whereby a least significant digit of 5 is rounded down. The *gpart* function uses mathematical rounding, meaning that 5 rounds up.

Installing *gmath*

The installation process will depend on the version of Excel used. The following notes relate to Excel 2010 on a Windows 10 PC, but I expect the process to be similar on other configurations.

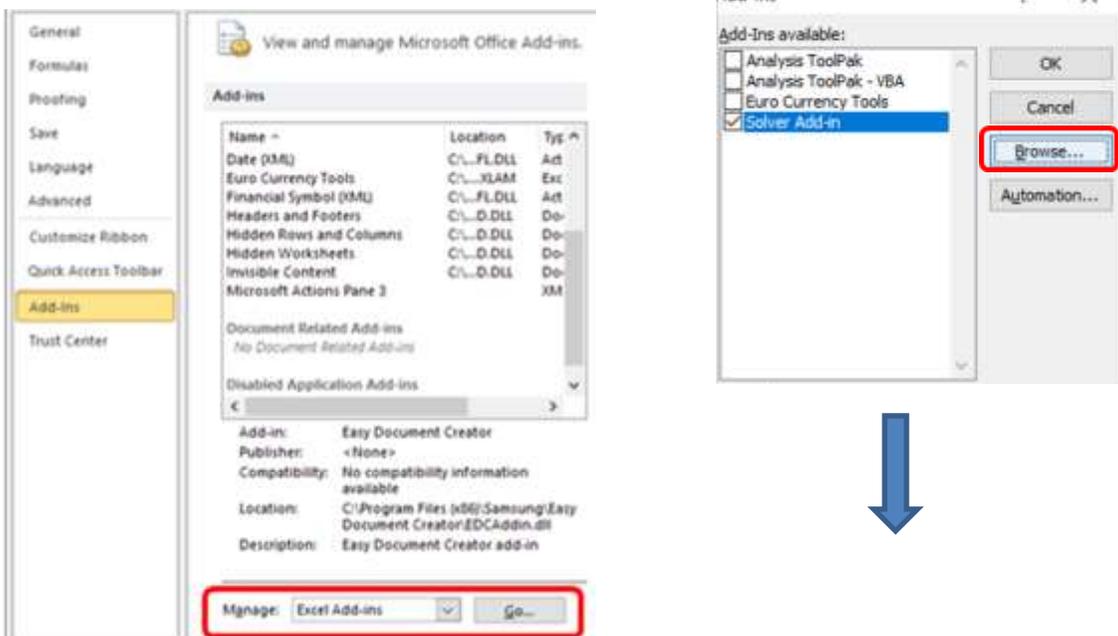
First, visit <http://www.va7lel.ca/sundial/gmath.xlam> to obtain the file. Save it to a convenient temporary location. Using Windows Explorer or the Desktop view, also copy the file to your clipboard.

Now start Excel and choose *File / Options* from the menu. On the page that pops up, choose *Add-Ins*.

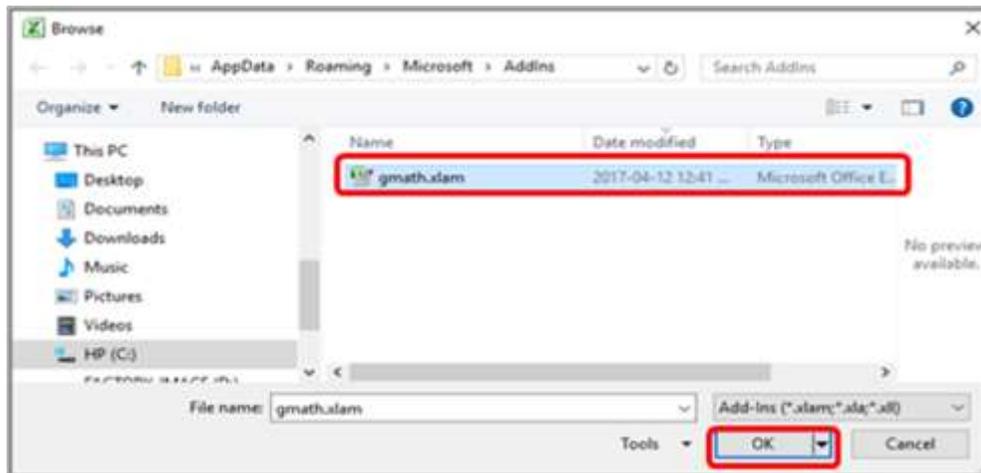


On the next pop-up window, choose *Manage Excel-Add-ins*, and click *Go*.

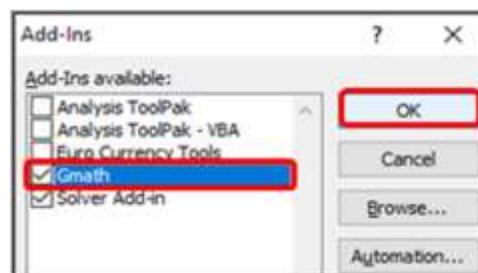
Then choose *Browse*.



When the next window appears paste the file from your clipboard, then click OK.



You will see *gmath* in your list of add-ins. Click OK to finish.



Should you ever wish to disable the *gmath* add-in, return to this list and remove the applicable check mark.

Depending on the security settings of your PC, you will probably need to tell Excel to trust the folder where you pasted the file (top image, this page). This is done using the *File / Options / Trust Center* menu. The folder is usually `C:\Users\username\AppData\Roaming\Microsoft\AddIns` where **username** is replaced by your actual username.

Inquiries

Please contact me if you have any inquiries or suggestions for enhancements to *gmath*, and be sure to report any problems encountered, via email to steve.lelievre.canada@gmail.com.

References

- Benett, G. (1982). The Calculation of Astronomical Refraction in Marine Navigation. *Journal of Navigation*, 35(2), 255–259.
- Meeus, J. (1998). *Astronomical Algorithms* (2nd ed.). Richmond, VA: Willmann-Bell, Inc.
- Sæmundsson, Þ. (1986). Astronomical Refraction. *Sky and Telescope*, 72(July), 70.